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Four-fermion production with RacoonWW†

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Abstract:

RacoonWW is an event generator for $\mathrm{e^+e^-} \to \mathrm{WW} \to 4\,\mathrm{fermions}(+\gamma)$ that includes full tree-level predictions for $\mathrm{e^+e^-} \to 4f$ and $\mathrm{e^+e^-} \to 4f + \gamma$ as well as $\mathcal{O}(\alpha)$ corrections to $\mathrm{e^+e^-} \to 4f$ in the so-called double-pole approximation. We briefly sketch the concept of the calculation on which this generator is based and present some numerical results.

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Abstract. RacoonWW is an event generator for $e^+e^- \to WW \to 4\,\mathrm{fermions}(+\gamma)$ that includes full tree-level predictions for $e^+e^- \to 4f$ and $e^+e^- \to 4f+\gamma$ as well as $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to 4f$ in the so-called double-pole approximation. We briefly sketch the concept of the calculation on which this generator is based and present some numerical results.

1. Introduction

At LEP2 and future e^+e^- linear colliders, the most important processes to study the properties of the W boson are $e^+e^- \to WW \to 4f$. For integrated quantities the accuracy typically reaches the order of 1% at LEP2 and will even exceed the per-cent level at future colliders. To account for this precision in predictions is a non-trivial task.

High-precision calculations for four-fermion production are complicated for various reasons. At the aimed accuracy of some 0.1%, a pure on-shell approximation for the W bosons is not acceptable, i.e. the W bosons have to be treated as resonances. Since the description of resonances necessarily goes beyond a fixed-order calculation in perturbation theory, problems with gauge invariance occur. Discussions of this issue can be found in Refs. [2, 3]. A second complication arises from the need to take into account electroweak radiative corrections beyond the universal corrections. The full treatment of the processes $e^+e^- \to 4f$ at the one-loop level is of enormous complexity and involves severe theoretical problems with gauge invariance; up to now such results have not been published.

Since lowest-order calculations for $e^+e^- \to 4f$ have already been extensively discussed in the literature (see e.g. Ref. [4] and references therein), we concentrate on radiative corrections in the following. Recent results of a full lowest-order calculation for real photon emission, $e^+e^- \to 4f+\gamma$ are reviewed in Section 2. In Section 3 we summarize the strategy and present some results of the event generator RacoonWW, which combines full tree-level predictions for $e^+e^- \to 4f$ and $e^+e^- \to 4f+\gamma$ with an approximation for the virtual $\mathcal{O}(\alpha)$ corrections to four-fermion production via a resonant W-boson pair, $e^+e^- \to WW \to 4f$.

$\sigma/\,\mathrm{fb}$	$\sqrt{s}=$	189 GeV	500 GeV	2 TeV	10 TeV
$\mathrm{e^+e^-} \to \mathrm{u\,\bar{d}}\,\mu^-\bar{\nu}_\mu\,\gamma$	constant width	224.0(4)	83.4(3)	6.98(5)	0.457(6)
	running width	224.6(4)	84.2(3)	19.2(1)	368(6)
	complex mass	223.9(4)	83.3(3)	6.98(5)	0.460(6)
$\mathrm{e^+e^-} \to \mathrm{u\,\bar{d}\,e^-}\bar{\nu}_\mathrm{e}\,\gamma$	constant width	230.0(4)	136.5(5)	84.0(7)	16.8(5)
	running width	230.6(4)	137.3(5)	95.7(7)	379(6)
	complex mass	229.9(4)	136.4(5)	84.1(6)	16.8(5)

Table 1. Comparison of different finite-width schemes (taken from Ref. [5])

2. Full tree-level predictions for $\mathrm{e^+e^-} \to 4f + \gamma$

The processes $\mathrm{e^+e^-} \to 4f + \gamma$ are interesting mainly for two reasons. On the one hand, they are a source of (real) $\mathcal{O}(\alpha)$ corrections to gauge-boson pair production with a four-fermion final state. On the other hand, they are sensitive to anomalous quartic gauge-boson couplings, such as $\gamma\gamma$WW, γZWW, and $\gamma\gamma$ZZ. In the following we briefly summarize some results of Ref. [5], where the building block of the event generator RacoonWW [6, 7] is described that calculates cross sections for $\mathrm{e^+e^-} \to 4f + \gamma$ with arbitrary massless fermions.

In this event generator different schemes for treating gauge-boson widths are implemented. A comparison of results obtained in these different schemes is useful in order to get information about the size of gauge-invariance-breaking effects, which are present in some finite-width schemes. Table 1 contains some results on the total cross section for two semi-leptonic four-fermion final states and a photon, evaluated with different finite-width treatments. Similar to the case without photon emission, the SU(2)-breaking effects induced by a running width render the predictions totally wrong in the TeV range. For a constant width such effects are suppressed, as can be seen from a comparison with the results of the complex-mass scheme, which exactly preserves gauge invariance.

Figure 1 shows the photon-energy spectra for some typical four-fermion final states that correspond to WWγ production. Apart from the usual soft-photon pole, the spectra contain several threshold and peaking structures that are caused by photon emission from the initial state. The two relevant classes of diagrams are illustrated in Figure 2. Diagrams with the structure of Figure 2a correspond to triple-gauge-boson-production subprocesses and yield dominant contributions as long as the two virtual gauge bosons can become simultaneously resonant. For instance, WWγ production is dominant for $E_\gamma < 26.3\,\mathrm{GeV}$ (224 GeV) for a centre-of-mass (CM) energy of 189 GeV (500 GeV). The diagrams of Figure 2b correspond to γZ production with a subsequent four-particle decay of the resonant Z boson mediated by a soft photon or gluon V_3. Owing to the two-particle kinematics of γZ production such contributions lead to peaking structures around a fixed value of E_γ, which is located at 72.5 GeV (242 GeV) for a CM energy of

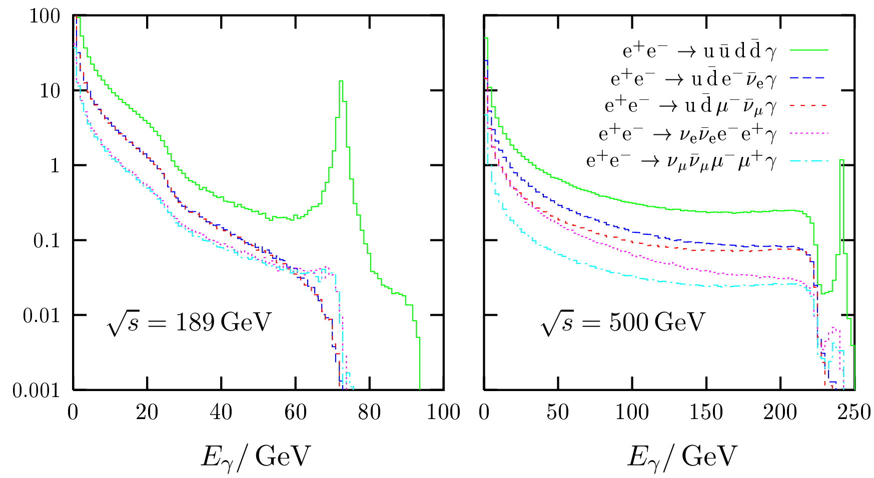

Figure 1. Photon-energy spectra $(\mathrm{d}\sigma/\mathrm{d}E_\gamma)/(\,\mathrm{fb}/\,\mathrm{GeV})$ for several processes (taken from Ref. [5])

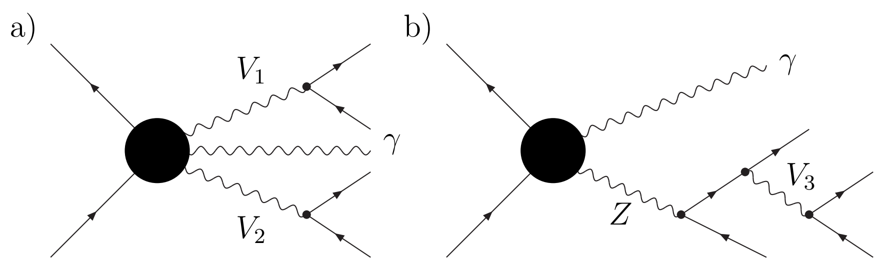

Figure 2. Diagrams for important subprocesses in $4f+\gamma$ production ($V_1, V_2 = \mathrm{W}, \mathrm{Z}, \gamma$, $V_3 = \gamma, \mathrm{g}$)

189 GeV (500 GeV).

Table 1 and Figure 1 also illustrate the effect of background diagrams, since final states that are related by the interchange of muons and electrons differ only by background diagrams. While the impact of background diagrams is of the order of some per cent for CM energies around 200 GeV, there is a large effect of background contributions already at 500 GeV. The main effect is due to forward-scattered $\mathrm{e}^\pm$, which is familiar from the results on $\mathrm{e^+e^-} \to 4f$. More numerical results for $\mathrm{e^+e^-} \to 4f + \gamma$ can be found in Refs. [5, 8].

3. Electroweak radiative corrections to $\mathrm{e^+e^-} \to \mathrm{WW} \to 4f$

3.1. Relevance of electroweak corrections

In the past, Monte Carlo generators for off-shell W-pair production (see e.g. Ref. [4]) typically included only universal electroweak $\mathcal{O}(\alpha)$ corrections such as the running of the electromagnetic coupling, $\alpha(q^2)$, leading corrections entering via the ρ-parameter, the Coulomb singularity, which is important near threshold, and mass-singular logarithms

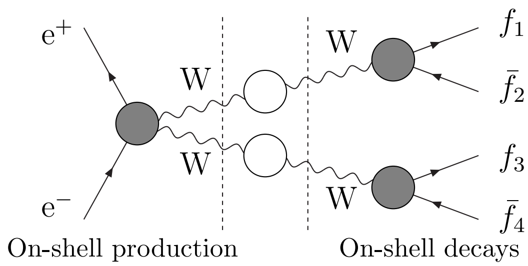

Figure 3. Diagrammatic structure of virtual factorizable corrections to $e^+e^- \to WW \to 4f$

$\alpha \ln(m_e^2/Q^2)$ from initial-state radiation, where Q^2 is not determined and has to be set to a typical scale for the process under consideration. The size of the remaining $\mathcal{O}(\alpha)$ contributions is estimated by inspecting on-shell W-pair production, for which the exact $\mathcal{O}(\alpha)$ correction and the leading contributions were given in Refs. [9] and [10], respectively. The difference $\delta_{\mathrm{IBA}} - \delta$ between an "improved Born approximation" δ_{IBA}, which is based on the above-mentioned universal corrections, and the corresponding full $\mathcal{O}(\alpha)$ correction δ corresponds to the non-leading corrections and has already been discussed in Refs. [2, 11]. For total cross sections this difference amounts to $\sim$ 1–2% for LEP2 energies, but to $\sim$ 10–20% in the TeV range; for distributions the difference is even larger in general. Thus, in view of a desired accuracy of some 0.1%, the inclusion of non-leading corrections is indispensable.

3.2. Electroweak corrections in double-pole approximation

Fortunately, the full off-shell calculation for the processes $e^+e^- \to WW \to 4f$ in $\mathcal{O}(\alpha)$ is not needed for most applications. Sufficiently far above the W-pair threshold a good approximation can be obtained by taking into account only those contributions that are enhanced by two resonant W bosons. The uncertainty from neglecting corrections to background diagrams can be estimated to some 0.1%, at least in the absence of special enhancement effects such as forward-scattered $e^\pm$. Doubly-resonant corrections to $e^+e^- \to WW \to 4f$ can be classified into two types [2, 12, 13]: factorizable and non-factorizable corrections.

Factorizable corrections are those that correspond either to W-pair production or to W decay. We first focus on virtual factorizable corrections, which are represented by the schematic diagram of Figure 3, in which the shaded blobs contain all one-loop corrections to the production and decay processes, and the open blobs include the corrections to the W propagators. The corresponding matrix element is of the form

$$\mathcal{M} = \underbrace{\frac{R_{+-}(k_+^2,k_-^2)}{(k_+^2-M_W^2)(k_-^2-M_W^2)}}_{\text{doubly-resonant}} + \underbrace{\frac{R_{+}(k_+^2,k_-^2)}{k_+^2-M_W^2} + \frac{R_{-}(k_+^2,k_-^2)}{k_-^2-M_W^2}}_{\text{singly-resonant}} + \underbrace{N(k_+^2,k_-^2)}_{\text{non-resonant}}, \tag{1}$$

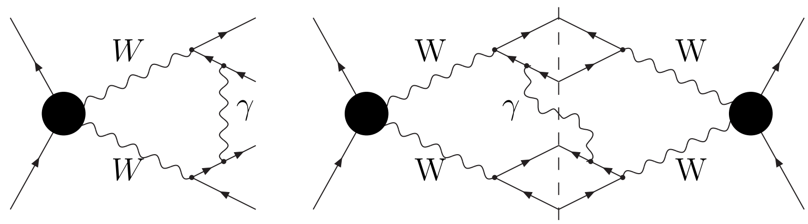

Figure 4. Examples of virtual and real non-factorizable corrections to $e^+e^- \rightarrow WW \rightarrow 4f$

and the double-pole approximation (DPA) amounts to the replacement

$$\mathcal{M} \rightarrow \frac{R_{+-}(M_W^2, M_W^2)}{(k_+^2 - M_W^2 + iM_W\Gamma_W)(k_-^2 - M_W^2 + iM_W\Gamma_W)}, \tag{2}$$

where the originally gauge-dependent numerator $R_{+-}(k_+^2, k_-^2)$ is replaced by the gauge-independent residue $R_{+-}(M_W^2, M_W^2)$ [12, 14]. The one-loop corrections to this residue can be deduced from the known results for the pair production [9] and the decay [15] of on-shell W bosons.

The formulation of a consistent DPA for the real corrections, and thus a splitting into factorizable and non-factorizable parts, is also possible, but non-trivial. The main complication originates from the emission of photons from the resonant W bosons. A diagram with a radiating W boson involves two propagators with momenta that differ only by the momentum of the emitted photon. If the photon momentum is large ($E_\gamma \gg \Gamma_W$), the resonances of these two propagators are well separated in phase space, and their contributions can be associated with photon radiation from exactly one of the production or decay subprocesses. For soft photons ($E_\gamma \ll \Gamma_W$), the resonances coincide, and the DPA is identical to the one without photon. However, for $E_\gamma \sim \Gamma_W$ the two resonance factors for the radiating W boson overlap. Although it is possible to decompose these factors into contributions associated with the subprocesses, a reliable estimate of the accuracy of the corresponding DPA is not obvious.

Non-factorizable corrections comprise all those doubly-resonant corrections that are not yet contained in the factorizable ones, i.e. they include all diagrams involving particle exchange between the subprocesses. These corrections do not contain the product of two independent Breit–Wigner-type resonances for the W bosons, i.e. the production and decay subprocesses are not independent in this case. Simple power-counting arguments reveal that such diagrams only lead to doubly-resonant contributions if the exchanged particle is a photon with energy $E_\gamma \lesssim \Gamma_W$; all other non-factorizable diagrams are negligible in DPA. Two relevant diagrams are shown in Figure 4, where the full blobs represent tree-level subgraphs. We note that diagrams involving photon exchange between the W bosons contribute both to factorizable and non-factorizable corrections; otherwise the splitting into those parts would not be gauge-invariant. The calculation of non-factorizable corrections to $e^+e^- \rightarrow WW \rightarrow 4f$ was discussed in Refs. [16–18] in detail. A numerical discussion of the sum of virtual and real non-factorizable corrections can be found in Refs. [17, 18].

3.3. Results for $\mathcal{O}(\alpha)$ corrections in double-pole approximation

Different versions of DPA have already been used in the literature [19–22]. For instance, following a semi-analytical approach, a consistent application of the DPA for the virtual and real corrections to four-lepton production was presented in Ref. [19]. In such an approach, however, it is not possible to apply all experimentally relevant phase-space cuts, and effects of recombining photons with nearly collinear charged fermions cannot be treated realistically. For the latter reason, in Ref. [19] the invariant masses of the W bosons were defined strictly in terms of the invariant masses of the corresponding decay fermion pairs. As a result, the invariant-mass distributions in $M_\pm = \sqrt{k_\pm^2}$ received large corrections from final-state radiation, namely $-20\,\mathrm{MeV}$, $-39\,\mathrm{MeV}$, and $-77\,\mathrm{MeV}$ for $\tau^+\nu_\tau$, $\mu^+\nu_\mu$, and $\mathrm{e}^+\nu_\mathrm{e}$ final states at $\sqrt{s} = 184\,\mathrm{GeV}$, respectively. These results have been qualitatively confirmed by YFSWW in Ref. [21], where the $\mathcal{O}(\alpha)$ corrections to W-pair production [20] were supplemented by final-state radiation in a leading-log approach. Note, however, that the large shifts are due to mass-singular logarithms like $\alpha\ln(m_l/M_\mathrm{W})$, which occur because no recombination of the fermions with collinear photons is performed. More realistic definitions of $k_\pm^2$, which have to include photon recombination, effectively replace the mass-singular logarithms by logarithms of a minimum opening angle for collinear photon emission. This expectation was also confirmed in Ref. [21].

At present, RacoonWW [6, 7] is the only Monte Carlo event generator with a complete implementation of the $\mathcal{O}(\alpha)$ corrections to $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW} \to 4f$ in DPA. In RacoonWW, only the virtual corrections are treated in DPA, whereas the real-photonic corrections are based on the full lowest-order calculation for $\mathrm{e}^+\mathrm{e}^- \to 4f + \gamma$ described in Ref. [5]. This means, in particular, that only the virtual corrections are split into factorizable and non-factorizable corrections. Note that this approach requires a careful treatment of IR and mass singularities, since the singularity structure in the virtual and real corrections are related to different lowest-order cross sections. For the virtual corrections the DPA Born cross section for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW} \to 4f$ is relevant, while for the real corrections the full Born cross section for $\mathrm{e}^+\mathrm{e}^- \to 4f$ applies. In RacoonWW one can choose between two different methods for the matching between virtual and real corrections; one method is based on phase-space slicing, the other on the subtraction method described in Ref. [23]. More details about the RacoonWW approach can be found in Refs. [6, 7].

Figures 5 and 6 show some results of RacoonWW for the semileptonic reaction $\mathrm{e}^+\mathrm{e}^- \to \nu_\mu\mu^+\mathrm{d}\bar{\mathrm{u}}$ at the typical LEP2 CM energy 200 GeV, where θ_W is the W production angle (with the W momentum being defined by the total momentum of ν_μ and μ^+) and M_du is the invariant mass of the quark pair. The precise definition of the input and of the photon-recombination procedure, as well as a detailed discussion of more results, can be found in Ref. [6]. For the photon recombination we first determine the lowest invariant mass $M_{\gamma f}$ built by an emitted photon and a charged final-state fermion. If $M_{\gamma f}$ is smaller than a fixed recombination cut M_rec the photon momentum is added to

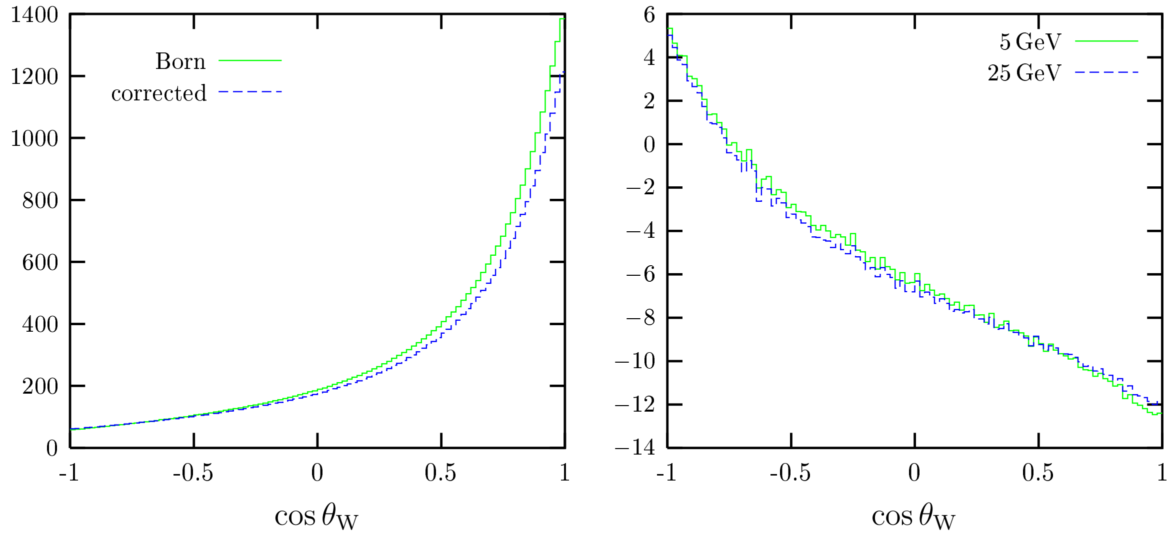

Figure 5. Production-angle distribution $(\mathrm{d}\sigma/\mathrm{d}\cos\theta_{\mathrm{W}})/\,\mathrm{fb}$ (left) and relative correction $\delta/\%$ (right) for $\mathrm{e^+e^-} \to \nu_\mu\mu^+\mathrm{d\bar{u}}$ at $\sqrt{s} = 200\,\mathrm{GeV}$ (taken from Ref. [6])

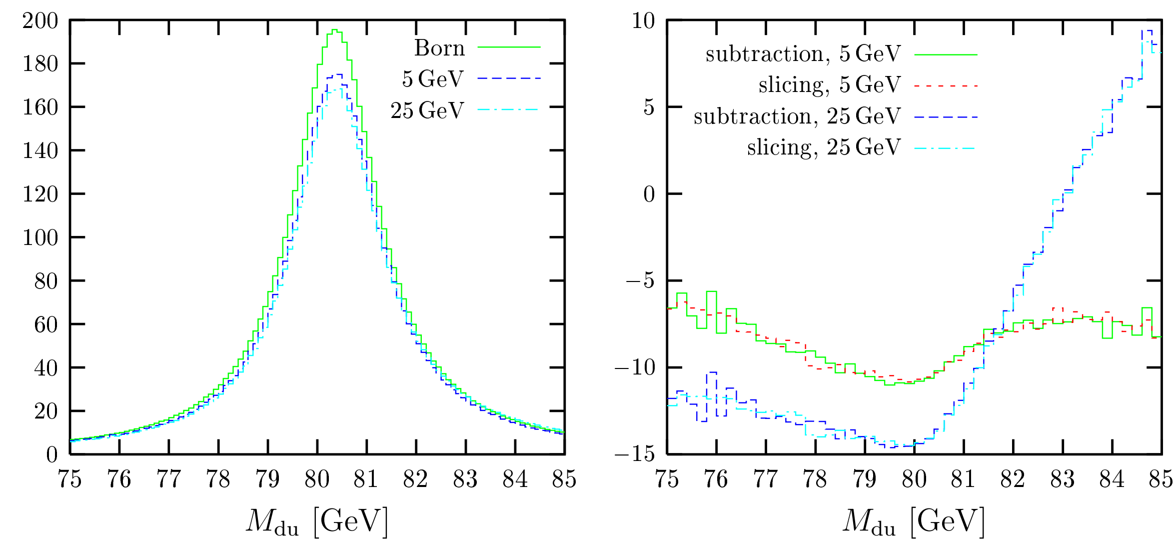

Figure 6. Invariant-mass distribution $(\mathrm{d}\sigma/\mathrm{d}M_{\mathrm{du}})/(\,\mathrm{fb}/\,\mathrm{GeV})$ of the quark pair (left) and relative correction $\delta/\%$ (right) for $\mathrm{e^+e^-} \to \nu_\mu\mu^+\mathrm{d\bar{u}}$ at $\sqrt{s} = 200\,\mathrm{GeV}$ (taken from Ref. [6])

the one of the corresponding fermion f. The curves denoted by "5 GeV" and "25 GeV" correspond to the respective values of M_{rec}. While the production-angle distribution is not very sensitive to M_{rec}, the invariant-mass distribution strongly depends on the recombination procedure, as expected from the above discussion. For different recombination procedures the maxima of the line shapes differ by up to 30–40 MeV [6]. As can be seen from Figure 6, there is a tendency to shift the maxima to larger invariant masses if more and more photons are recombined.

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